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## One, Two, and Three Line Segment "Straw Man" Models

 Soybean Yields in Iowa, Illinois, and IndianaJeanne L. Sebaugh

COMPARISON OF ONE, TWO, AND THREE LINE SEGMENT "STRAW MAN" MODELS FOR SOYBEAN YIELDS IN IOWA, ILLINOIS, AND INDIANA. By Jeanne L. Sebaugh; Statistical Research Division, Economics and Statistics Service, U.S. Department of Agriculture, Columbia, Missouri 65201; May 1981.
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#### Abstract

All of the straw man models attempt to explain differences in yields over time by fitting trend lines to the yield data. The one line segment straw man model, simple linear regression, describes a uniform increase in yields over time. The two and three line segment straw man models allow the rate of change in yields to vary over the time period. The performance of the three models in predicting soybean yields in Iowa, Illinois, and Indiana is compared based on eight model characteristics. There is little difference between the three models in relation to seven of the characteristics: objectivity, consistency with scientific knowledge, adequacy, timeliness, minimum costs, simplicity, and accurate current measures of modeled yield reliability. The one line model performs somewhat better than the other straw man models on the remaining characteristics, yield indication reliability.


Key words: Model comparison, yield modeling, linear regression.


## ACKNOWLEDGMENTS

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# COMPARISON OF ONE, TWO, AND THREE LINE SEGMENT "STRAW MAN" MODELS FOR SOYBEAN YIELDS IN IOWA, ILLINOIS, AND INDIANA 

## by

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#### Abstract

This research was conducted as part of the AgRISTARS Yield Model Development Project. It is part of task 3 (subtask 2) in major project element number 1 , as identified in the 1980 Yield Model Development Project Implementation Plan. As an internal project document, this report is identified as shown below.


AgRISTARS<br>Yield Model Development Project

YMD-1-3-2(81-05.1)

Development and application of techniques for crop yield model test and evaluation are important parts of the yield Model Development Project in AgRISTARS. 1\% Promising yield models available in the literature or from various researchers will be subjected to performance test and evaluation. In order that there may be a common reference for describing the capabilities and limitations of these models, criteria for doing so have been developed and described in a document entitled crop Yield Model Test and Evaluation Criteria (Wilson, et al., 1980). These criteria are used both in the evaluation of a single model and in comparisons between models.

The purpose of this document is to gain some experience in the application of the criteria for comparison purposes. Previous documents have used the same criteria for model evaluation. In addition, statistical tests which compare the performance of two models have been developed and are used. It is anticipated that the evaluation and comparison of other models will be done in a similar manner.

The models to be evaluated and compared were chosen to be quite simple since the focus of attention is on the "pilot test" of the procedures. The models involved are the "straw man" crop yield models developed and discussed by Kestle (1981). This document compares the one, two, and three line segment models which all regress yield on year.

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I) AgRISTARS (Agriculture and Resources Inventory Surveys Through Aerospace Remote Sensingl is a multi-agency research program to meet some current and new information needs of the U. S. Department of Agriculture.

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# Comparison of One, Two, and Three Line Segment <br> "Straw Man" Models for Soybean Yields in Iowa, Illinois, and Indiana 

Jeanne L. Sebaugh

## SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

Straw man model 1 , simple linear regression of yield over time, is preferred over the two and three line straw man models for predicting soybean yields. The one line model performs better than the other straw man models in terms of yield indication reliability. There is little difference betwéen the three models in relation to the other seven criteria. However, it can not be concluded from this analysis that the rate of change in yields has in fact been uniform over time. Also, other models not considered here may outperform straw man model 1.

## APPLICATION DESCRIPTION

Exercise in Applying Procedures
for Model Comparison
Test and evaluation of candidate crop yield models for use with particular crops and geographic regions are major tasks within the AgRISTARS program. In order that there may be common reference for describing the capabilities and limitations of competing models, criteria for crop yield model test and evaluation have been developed (Wilson, et al., 1980). In addition, the cooperative agreement with the Department of Statistics at the University of Missouri-Columbia has produced some related documents (Bhattacharyay, 1980; Moeschberger, 1980; Thompson, 1980).

This document describes the application of the criteria and statistical procedures to straw man models developed for soybeans in Iowa, Illinois, and Indiana (Kestle, 1981). It is hoped that this exercise will provide an opportunity to see how well the criteria and statistical procedures perform in actual use as well as establish some base lines for their implementation.

## REVIEW OF MODELS

## Straw Man Models Describe Technological Trends

All of the straw man models attempt to explain differences in crop yields over time by simply fitting trend lines to the yield data. Improvements in technology, including varieties, hybrids, fertilizers, insecticides, herbicides, farming practices, equipment, etc., have resulted in steady improvements in yields. There are occasional set backs, primarily due to weather, but the overall trend has been towards increasing yields. By using year as the only
independent variable, the straw man models demonstrate how much of the year-to-year differences in yield can be explained by this analogue to technological trend.

## Straw Man Mode1 1 - Uniform Trend Over Time

Straw man model 1 is a simple linear regression over time. The statistical model is

$$
E(Y)=\beta_{0}+\beta_{1} X,
$$

where $Y$ is the yield in quintals per hectare and $X$ is the corresponding year number ( $1950=0$ ).

The inherent assumption in a simple linear regression model is that the rate of change in the $Y$ variable is constant over the entire range of the $X$ values. In our case, this means that the year-to-year increases in yield are assumed to be the same throughout the entire time period. More information about straw man model 1 may be found in Evaluation of "Straw Man" Model 1, the Simple Linear Model. For Soybean Yields in Iowa, Illinois, and Indiana (Sebaugh, 1981).

## Straw Man Model 2 - Two Trends Over Time

Straw man model 2 consists of two intersecting line segments. The statistical model is

$$
\begin{aligned}
& E(Y)=\beta_{10}+\beta_{11} X, X \leq \tau \\
& E(Y)=\beta_{20}+\beta_{21} X, X>\tau,
\end{aligned}
$$

where $Y$ is the yield in quintals per hectare, $X$ is the corresponding year number $(1950=0)$, and $\tau$ is the join point where the two lines intersect. $\tau$ is unknown but is objectively estimated using a FORTRAN program based on Hudson's (1966) least squares algorithm. Following Kestle's (1981) suggestion, the slopes of the two lines, $\beta_{11}$ and $\beta_{21}$ are constrained to be positive and the last line segment is constrained to cover at least five years. The length of the first line segment is not so constrained since the beginning point is arbitrary.

This model assumes that $Y$ increases in a continuous fashion but that the rate of change in $Y$ is not the same over the entire range of $X$ values. Two different rates of change are allowed, $\beta_{11}$ and $\beta_{21}$. So year-to-year increases in yield may occur at a faster (or slower) rate later on in the time period than they did earlier in the time period. More information about straw man model 2 may be found in Evaluation of "Straw Man" Model 2, Two Trends Over Time, For Soybean Yields in Iowa, Illinois, and Indiana (Sebaugh, 1981).

Straw man model 3 consists of three intersecting line segments. The statistical model is

$$
\begin{aligned}
& E(Y)=\beta_{10}+\beta_{11} X, X \leq \tau_{1} \\
& E(Y)=\beta_{20}+\beta_{21} X, \tau_{1}<X \leq \tau_{2} \\
& E(Y)=\beta_{30}+\beta_{31} X, X>\tau_{2},
\end{aligned}
$$

where $Y$ is the yield in quintals per hectare, $X$ is the corresponding year number ( $1950=0$ ) , $\tau_{1}$ is the join point where the first two lines intersect, and $\tau_{2}$ is the join point where the last two lines intersect. $\tau_{1}$ and $\tau_{2}$ are unknown but are objectively estimated using a FORTRAN program based on Hudson's (1966) least squares algorithm. Again, following Kestle's (1981) suggestions, the slopes of the three lines, $\beta_{11}, \beta_{21}$, and $\beta_{31}$, are constrained to be positive and the last two line segments are constrained to cover at least five years each. The length of the first line segment is not so constrained since the beginning point is arbitrary.

This model assumes that $Y$ increases in a continuous fashion but that the rate of change in $Y$ is not the same over the entire range of $X$ values. Three different rates of change are allowed, $\beta_{11}, \beta_{21}$, and $\beta_{31}$. So year-to-year increases in yield may occur at different rates at different intervals over the time period. More information about straw man model 3 may be found in Evaluation of "Straw Man" Model 3, Three Trends Over Time, For Soybean Yields in Iowa, Illinois, and Indiana (Sebaugh, 1981).

COMPARISON METHODOLOGY

## Eight Model Characteristics to be Compared

The document, Crop Yield Model Test and Evaluation Criteria, (Wilson, et al., 1980), states:
> 'The model characteristics to be emphasized in the evaluation process are: yield indication reliability, objectivity, consistency with scientific knowledge, adequacy, timeliness, minimum costs, simplicity, and accurate current measures of modeled yield reliability."

The models will be compared using these characteristics. Each characteristic is discussed individually without regard to the other characteristics. The present discussion makes no presumption as to the relative importance of the characteristics.

## Quantitative Model Comparisons <br> Are Based on the Same Data

Direct quantitative comparisons between models will be made for two of the previously mentioned criteria: (1) yield indication reliability and (2) accurate current measures of modeled yield reliability. The quantities
involved are derived from the observed yields and the model predicted yields and standard errors of prediction obtained from independent bootstrap tests for each of ten years (1970-1979). The same base period is used for all models in computing model related values for a particular year.

The average production and yield over the ten year test period are listed in Table 1 for each geographic area. Also shown is the percent production each crop reporting district (CRD) contributes to its state and the three state region and the percent production each state contributes to the region. The percentage of regional production for each CRD is shown graphically in Figure 1. Darker shades indicate higher productivity.

Separate models are derived for each CRD, state, and the region. Model related values (predictions and standard errors of prediction) at the state level are also obtained by using a weighted average of that state's CRD model values. Model related values for the region are also obtained using a weighted average of the values from the CRD models and from the state models. The weighting factor used is harvested acreage. Results obtained by aggregating from the CRD models are identified as "CRD aggr." Results obtained by aggregating from the state models are identified as "states aggr."

## Models Are Ranked According to Performance

Models are ranked for each of the following indicators of yield reliability (order does not imply relative importance):
(1) the bias,
(2) the root mean square error (RMSE),
(3) the standard deviation (SD),
(4) the percent of years the absolute value of the relative difference exceeds ten percent,
(5) the largest absolute value of the relative difference,
(6) the next largest absolute value of the relative difference,
(7) the percent of years in which the direction of change from the previous year in the $Y^{\prime}$ 's agrees with the $Y^{\prime}$ s.
(8) the percent of years in which the direction of change from the average of the previous three years in the $\hat{Y}^{\prime}$ s agrees with the $Y^{\prime} s$, and
(9) the Pearson correlation coefficient between the actual and predicted yields during the independent test years.

Models are also ranked according to the value of the Spearman correlation coefficient which indicates the utility of the model's current measure of modeled yield reliability. For most of the indicators (1-6), the model with the smallest numeric value exhibits the best performance in terms of yield

TABLE 1
AVERAGE PRONUCTION AND YIELT
IOWA. ILLOYBEANS INIIIANA

| STATE | CRD | PRODUCTION ouintals | $\begin{aligned} & (1: 000) \\ & \text { BUSHELS } \end{aligned}$ | $\begin{aligned} & \text { PERCFNT OF } \\ & \text { STATE REGION } \end{aligned}$ |  | $\begin{aligned} & \text { YIFLD } \\ & \text { OVTL/HA BU/ACRE } \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IOWA |  |  | 39,439 | 9 |  |  |  |
|  | 20 | 10,734 | 40,439 | 17.3 | 6.4 | 22.7 | 34.8 33.8 |
|  | 30 | 3,929 | 14.435 | 6.2 | 2.3 | 21.7 | 32.3 |
|  | 40 | 8.189 | 30.090 | 12.9 | 4.3 | 22.3 | 33.1 |
|  | 50 | 11.207 | 41.177 | 17.7 | 6.5 | 23.7 | 35.3 |
|  | 90 | 4.996 5.016 | 18.358 18.430 | 7 7 9 | 2.9 | 24.5 22.1 | 36.4 32.9 3 |
|  | 80 | 3,107 | 11.415 | 4.9 | 1:9 | 20.4 | 30.4 |
|  | 90 | 5.187 | 19,060 | 8.2 | 3.0 | 23.1 | 34.3 |
| state |  | 63.357 | 232,773 |  | 36.8 | 22.9 | 34.0 |
| ILLINOIS | 10 | 5,670 | 20,834 | 7.5 | 3.3 | 24.0 | 35.6 |
|  | 20 | 6:960 | 25:575 | 9.2 | 4.0 | $22 . ?$ | 33.0 |
|  | 30 | 6.331 | 23.263 | 8.4 | 3.7 |  | 35.0 |
|  | 40 | 10.855 | 39,885 | 14.4 | $6 \cdot 3$ | 25.0 | 37.2 |
|  | 50 | 12.870 | 47,288 | 17.1 | 7.5 | 24.2 | 36.0 |
|  | 780 | 11,739 | 41.931 | 15.6 | 6.8 | 20. 8 | 34099 |
|  | 80 | 4.800 | 17.637 | 6.4 | 2.8 | 19.2 | 2 C .6 |
|  | 90 | 4.694 | 17.248 | 6.2 | 2.7 | 17.4 | 25.8 |
| state |  | 75,333 | 276.795 |  | 43.7 | 22.4 | 33.3 |
|  |  |  |  |  |  |  |  |
| INDIANA |  |  |  |  |  |  |  |
|  | 20 | 3,717 | 13.659 | 11.1 | $2 \cdot 2$ | 21.5 | 32.0 |
|  | 30 | 3,897 | 14.319 | 11.6 | 2.3 | 20.8 | 31.0 |
|  | 40 | 4,443 | 16.326 | 13.2 | 2.5 | 22.5 | 33.5 |
|  | 50 60 | 8.100 $3.14 ?$ | 29.761 | $24 \cdot \frac{1}{9}$ | 4.7 | 23.6 | 35.1 31.2 |
|  | 70 | 3.304 | 12.139 | $9: 3$ | 1:9 | 21:0 | 31:3 |
|  | 80 | + 709 | 2.604 | $2 \cdot 1$ | $0 \cdot 4$ | 18.3 | 27:3 |
|  | 90 | 1.042 | 3,827 | 3. | 0.5 | 19.8 | 27.9 |
| STATE |  | 33,612 | 123,500 |  | 19.う | 21.9 | 32.5 |
|  |  |  |  |  |  |  |  |
| RFGION |  | 172,301 | 633,088 |  |  | 22.5 | 33.4 |

Figure 1. Production of soybeans by CRD (1970-79 average) as a percent of the regional total. Darker shades indicate CRDs with higher production.

reliability and is given a rank of 1 . For the remaining quantities, the model with the largest value exhibits the most desirable performance. If models are tied for the same level of performance, they are all assigned the lowest rank for which they are tied. For example, if two models are tied for best performance, they are both assigned a rank of 1 , the lower of ranks 1 and 2.

It should be remembered that the models are ranked only in relation to each other and not to an absolute standard. Therefore, saying that a particular model performs best or is superior to or more desirable than another model does not necessarily imply that the model is the best of all possible models. It is the best of only those with which it is currently being compared.

## Models are Compared Using Statistical <br> Tests Based on $d=\widehat{Y}-Y$

It is desirable to run a statistical test comparing the reliability of competing models. A formal statistical test considers the variability of model performance over time and allows the user to specify an upper limit on the probability of incorrectly declaring one model better than another. This probability is known as $\alpha$, the level of significance, or the Type I error.

However, because of the manner in which models are chosen for testing and how they are evaluated, it is challenging to construct a meaningful statistical test. Only yield models which have been presented in the literature or developed by known experts are considered. Therefore, a priori, great differences between the reliability of the models are not expected. A powerful statistical procedure is needed which is able to detect small, although important, differences in reliability. Also, the test should be able to function well with relatively small samples of data for each model, say ten years.

The test should also perform well when only two models are being compared. Often only two models of a particular type, for example, two monthly weather data models or two daily weather data models, are competitive and available for testing. When models of different types are to be compared, it is unlikely that all possible model comparisons will be made. It is more likely that the best models of each type will be compared.

It would appear that an $F$ test could be useful in comparing the mean square errors of two models. However, if the mean square errors are based on ten years of test data and $\alpha=.05$, then one model's mean square error must be four times larger than anothers before the models can be declared different. This is an unreasonable requirement since models which are in the evaluation process will almost always be more competitive than this.

A test may be constructed by considering that one model is considered more reliable than another model if its predicted yields, $\mathrm{Y}^{\prime}$ s, are closer to the actual yields, Y's. No difference in the reliability of two models for a particular year means that the absolute value of the difference between their predicted yields and the actual yield is the same. The absolute value of the difference is used because it does not matter whether one model overestimates and the other underestimates or whether they both over or underestimate. The reliability of a model for that year is related to the amount
of the discrepancy, not its direction. We may define $\left|d_{1}\right|=\left|\hat{Y}_{1}-Y\right|,\left|d_{2}\right|=$ $\left|\hat{Y}_{2}-Y\right|$, and $D=\left|d_{1}\right|-\left|d_{2}\right|$. Then the models are equatiy reliable in a year for which $D$ equals zero. If $D$ is not equal to zero, one model is more reliable than the other for that year. In formal terms, we want to test the null hypothesis that there is no difference in the reliability of the models over all years. To do so the values of $D$ from the ten test years may be used to compute a test statistic and a decision made whether or not to reject the null hypothesis. Since the results for the models are paired each year, paired-sample statistical tests are used.

Two types of paired-sample statistical tests are used: a parametric test using the student " $t$ " test statistic and a nonparametric test using the Wilcoxon signed rank test statistic. One reason for applying both tests is that they require different assumptions. The parametric t-test assumes the $D$ values are normally distributed while the nonparametric test does not. The d values may be considered to be approximately normally distributed. The $|d|$ values would then be folded normals rather than normally distributed. Although both models are folded at $|\mathrm{d}|=0$, their means may be different and the distribution of $D$ has a possibility of not being normally distributed. The t-test is robust with respect to the normality assumption; however, this possible violation of the assumption is one reason for also running the nonparametric test.

The other reason for running both tests concerns the conditions under which the null hypothesis is rejected by each test. Using the parametric test, the basis for rejecting the null hypothesis is the average size of the $D$ values as compared to their variability. The t-test statistic is the average of the sample D's divided by the sample standard error of the D's. The hypothesis will be rejected and the model with the smaller $|d|$ values declared more reliable if $t$ is large (either positive or negative). However, it is possible that one model could have a smaller $|d|$ value for each of the test years, in other words, be very consistent in outperforming the other model, and still the null hypothesis may not be rejected by the parametric test unless the average value of $D$ is large enough. The parametric test implicitly requires that one model have more years with smaller $|d|$ values than the other model and explicitly requires that, on the average, the $|d|$ values be smaller by a sufficient amount before that model may be declared more reliable.

Using the nonparametric test, the null hypothesis will always be rejected if one model has smaller $|\mathrm{d}|$ values for each of the test years, regardless of the magnitude of the $D$ values. Therefore, if the models are very competitive in terms of the $|d|$ values each year, but one model consistently, although slightly, outperforms the other model, the nonparametric test will still declare the consistent model to be more reliable.

The hypothesis of equal model performance will only be rejected by the nonparametric test if one model has more years with smaller $|\mathrm{d}|$ values than the other model. The model with more smaller $|\mathrm{d}|$ values is considered the more reliable model in terms of consistency of performance. However, to reject the null hypothesis and declare one model clearly better than another, consistency of performance is not a sufficient requirement (although it is necessary). Consider the situation in which one model is more consistent
than the other but the largest $D$ values occur when the less consistent model performs better. In the few years the less consistent model performs better, it performs much better. A dilemma exists since one model is more consistent than the other but the biggest differences between the models occur when the consistent model performs worse. The null hypothesis will not be rejected and the consistent model will not be declared better if this situation occurs. The null hypothesis will be rejected only if one model is more consistent and the biggest differences between the models occur when the consistent model performs better.

## MODEL COMPARISON

$\frac{\text { Indicators of Yield Reliability Based on }}{\mathrm{d}=\hat{\mathrm{Y}}-\mathrm{Y} \text { Show All Models Have Small Bias }}$
$\frac{\text { But the One Line Model Has the Smallest Root }}{}$
Mean Square Error and Standard Deviation

The model values and comparative ranks for the bias, the root mean square error (RMSE), and the standard deviation (SD) are given in Tables 2, 3 and 4. There is no clear cut best model in terms of bias. At the CRD level, the three line model has the smallest bias more of ten than either otier model ( 12 out of 27 times). However, it is not ranked 1 for bias at either the state or region levels. This inconsistency in performance is symptomatic of the small amount of bias produced by each of the models.

The one line model is clearly the most accurate model. In 24 of the 27 CRDs ( $89 \%$ ), the one line model has the smallest root mean square error and standard deviation. The best performing model in each CRD according to the root mean square error is shown in Figure 2. The one line model also has the best performance at the state and region levels.

The three line model has the worst accuracy. It has the largest RMSE in 15 ( $56 \%$ ) and the largest SD in 16 (59\%) out of 27 CRDs. The state and region results are also poorest for the three line model.

Indicators of Yield Reliability Based on $r d=(d / Y) * 100$ Show the One Line Model Performing Best

The model values and comparative ranks for the indicators of yield reliability based on the relative difference, rd, are given in Tables 5, 6 and 7. These indicators are valuable for demonstrating the worst performance of a model. Therefore, the best performing model will have the smallest values for the percent of years the absolute value of the relative difference exceeds ten percent and for the largest and the next largest absolute value of the relative difference.

In 20 of 27 CRDs ( $74 \%$ ), the one line model has the smallest (or is tied for the smallest) percent of years in which the absolute value of rd exceeds ten percent (Figure 3). Except for the Iowa state model, the one line model also performs best at the state and region levels. There is little difference between the performance of the two and three line models.


TABLE 3
YODEL COMOARISON GASEO ON THE
ROOT MEAN SQUARE ERROQ (RUINTALS/HECTARE)
DERIVED FROM INDEPENDENT TEST YEARS
STRAW YAN MODELS - SOYBEANS

| STATE CRD | RMSE LINE |  | 2 LINES |  | $\begin{aligned} & 3 \\ & \text { RMSE IINES } \\ & \text { RANIK } \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| IOWA 10 1 | 2.70 | (1) | 3.35 | (2) | 3.41 | (3) |
| 201 | 1.49 | (1) | 1.68 | (2) | 1.72 | (3) |
| 301 | 2.13 | (1) | 2.68 | (3) | 2.54 | (2) |
| 401 | 2.60 | (1) | 2.90 | $(3)$ | 2.88 | (2) |
| 501 | 2.60 | (1) | 2.74 | (2) | 3.02 | $(3)$ |
| 601 | 2.19 | (1) | 2.65 | (3) | 2.44 | (2) |
| 701 | 2.18 | (3) | 2.00 | (2) | 1.90 | (1) |
| 80 90 | 2.87 | (1) | 3.34 3.22 | (2) | 3.51 | (3) |
| 90 | 2.66 | (1) | 3.22 | (2) | 3.34 | (3) |
| STATE MODEL | 1.95 | (1) | 2.29 | (2) | 2.51 | (3) |
| CRDS AGGR. | 1.95 | (1) | 2.27 | (2) | 2.32 | (3) |
|  |  |  |  |  |  |  |
| ILLINOIS 101 | 2.93 | (1) | 3.41 | (7) | 3.46 | (3) |
| ILLINO 201 | 2.71 | (1) | 2.83 | (2) | 2.96 | (3) |
| 301 | 2.80 | (1) | 3.32 | (3) | 3.19 | (2) |
| 401 | 3.21 | (1) | 3.72 | (2) | 3.93 | $(3)$ |
| 501 | 2.74 | (1) | 3.31 | (2) | 3.34 | (3) |
| 601 | 2.49 | (1) | 2.75 | (3) | 2.65 | (2) |
| 701 | 2.47 | (1) | 2.67 | (3) | 2.94 | (3) |
| 801 | 2.39 | (1) | 2.40 | (2) | 2.46 | (3) |
| 90 | 2.44 | (1) | 2.45 | (2) | 2.95 | (3) |
|  |  |  |  |  |  |  |
| STATE MODEL. CRDS AGGR.I | $\begin{aligned} & 2.51 \\ & 2.53 \end{aligned}$ | $\left(\begin{array}{l} 1 \\ 1 \\ 1 \end{array}\right)$ | 2.69 2.80 | $\left(\begin{array}{l} 2) \\ (2) \end{array}\right.$ | $\begin{aligned} & 3.08 \\ & 2.90 \end{aligned}$ | $(3)$ $(3)$ |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| INDIANA $\begin{array}{lll}10 & 1 \\ & 20 & 1\end{array}$ | 2.23 2.65 | (1) | 2.62 | $\left(\begin{array}{l}2) \\ (3)\end{array}\right.$ | 2.85 2.99 | (3) |
| 10 30 | 2.65 2.34 | (1) | 2. 46 | (2) | 2.56 | (3) |
| 401 | 2.67 | (1) | 3.44 | (3) | 3.40 | (2) |
| 50 | 2.51 | (1) | 3.03 | (3) | 2.94 | (2) |
| 601 | 1.79 | (1) | 2.17 | (2) | 2.36 | (3) |
| 701 | 1.93 | (1) | 2.04 | (3) | 1.97 | (2) |
| 801 | 2.74 | (3) | 2.39 | (2) | $2 \cdot 19$ | (1) |
| 90 | 2.74 | (3) | 2.34 | (1) | 2.37 | (2) |
| STATE MODEL | 2.05 | (1) | 2.44 |  |  |  |
| CQDS $\triangle$ GGR. | 2.03 | (1) | 2.47 | $(2)$ | 2.50 | (3) |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 1.96 1.99 | (1) | 2.39 2.30 | (2) | 2.51 | (3) |
| STATES AGGR.I | 2.00 | (1) | 2.29 | (2) | 2.44 | $(3)$ |

Figure 2. Number indicates the model with smallest root mean square error for soybean yields based on test years 1970-1979. Darker shades indicate CRDs with higher production.


| TABLF 4 <br> UNDEL COMPARISON BASED ON THE STANDARD DFVIATION (OUINTALSHECIARE) DEKIVED FROM INDEPENDENT TEST YEARS |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| straw man models - soybeans IOWA. ILLINOIS. INDIANA |  |  |  |  |  |  |
| STATE CRD | $\begin{aligned} & \text { S I.INE } \\ & \text { SD } \end{aligned}$ |  | $\begin{aligned} & \text { MONEL } \\ & \text { SíLIVFS } \\ & \text { SANK } \end{aligned}$ |  | $\begin{gathered} 3 \\ S^{3} \text { LINEC } \\ \text { QATK } \end{gathered}$ |  |
| IOWA 10 |  |  | $\begin{aligned} & 3.35 \\ & 1: 67 \\ & 1.64 \\ & 2.89 \\ & 2.73 \\ & 1.65 \\ & 1.95 \\ & 3.32 \\ & 3.22 \end{aligned}$ | $\left.\begin{array}{l} (2) \\ (2) \\ (3) \\ (3) \\ 3 \\ 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ (2) \\ 21 \end{array}\right)$ | $\begin{aligned} & 3.41 \\ & 1.91 \\ & 2.53 \\ & 2.82 \\ & 3.01 \\ & 2.48 \\ & 1088 \\ & 3.60 \\ & 3.34 \end{aligned}$ | $\left.\begin{array}{l}\left(\begin{array}{l}3 \\ 3 \\ 3\end{array}\right. \\ (2) \\ (2) \\ 3 \\ 12 \\ (1) \\ (3) \\ (3) \\ 3\end{array}\right)$ |
| TOWA $\quad 20$ | 2.58 | (1) |  |  |  |  |
| 30 | 2.01 | (1) |  |  |  |  |
| 40 | 2.60 | (1) |  |  |  |  |
| 50 60 | 2.57 | (1) |  |  |  |  |
| 70 | 2.191 | (2) |  |  |  |  |
| 80 | 2.86 | (1) |  |  |  |  |
| 90 | 2.64 | (1) |  |  |  |  |
| STATE MODEL | 1.75 | (1) | 2.29 | (2) | $\begin{aligned} & 2.49 \\ & 2.32 \end{aligned}$ | $(3)$ |
| CRDS AGG2. | 1.95 | (1) | 2:27 | (2) |  |  |
| ILLINOIS $\begin{array}{r}10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \\ 90\end{array}$ | $\begin{aligned} & 2.91 \\ & 2.61 \\ & 2.74 \\ & 3.21 \\ & 2.72 \\ & 2.46 \\ & 2.47 \\ & 2.39 \\ & 2.44 \end{aligned}$ |  | $\begin{aligned} & 3.41 \\ & 2.83 \\ & 3.31 \\ & 3.71 \\ & 3.29 \\ & 2.74 \\ & 2.65 \\ & 2.39 \\ & 2.38 \end{aligned}$ | $\begin{aligned} & (2) \\ & (2) \\ & (3) \\ & (2) \\ & (2) \\ & (3) \\ & 12) \\ & (2) \\ & (1) \end{aligned}$ | $\begin{aligned} & 3.46 \\ & 2.84 \\ & 3.19 \\ & 3.91 \\ & 3.37 \\ & 2.66 \\ & 2.71 \\ & 2.46 \\ & 2.82 \end{aligned}$ | (3)3323331133333 |
|  |  |  |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (2) |  |  |  |  |
| STATE MODELCRDSGGGR. | $2 \cdot 51$ | (1) | 2.672.80 | (2) | 3.082.90 | $\left(\begin{array}{l}(3) \\ (3)\end{array}\right.$ |
|  |  | (I) |  |  |  |  |
| INDIANA $\begin{array}{lr}10 \\ & 20 \\ & 30 \\ & 40 \\ & 50 \\ & 60 \\ & 70 \\ & 80 \\ & 90\end{array}$ | $\begin{aligned} & 2.15 \\ & 2: 49 \\ & 2.06 \\ & 2.07 \\ & 2.34 \\ & 1: 60 \\ & 1: 83 \\ & \lambda: 12 \\ & 2: 21 \end{aligned}$ |  | $\begin{aligned} & 2.62 \\ & 3.09 \\ & 2.45 \\ & 3.41 \\ & 3.02 \\ & 2.03 \\ & 2.00 \\ & 2.21 \\ & 2.25 \end{aligned}$ | $\left.\begin{array}{l}(2) \\ (3) \\ (2) \\ 3 \\ 3 \\ 3 \\ (2) \\ (3) \\ 3 \\ 3 \\ 3 \\ 2\end{array}\right)$ | $\begin{aligned} & 2.84 \\ & 2.98 \\ & 2.56 \\ & 3.36 \\ & 2.87 \\ & 2.22 \\ & 1.97 \\ & 2.12 \\ & 2.30 \end{aligned}$ | $\begin{aligned} & \left(\begin{array}{l} 3 \\ 2 \\ 3 \\ 3 \\ (2) \\ 2 \\ 2 \\ 3 \\ 2 \\ 1 \\ 1 \\ (3) \end{array}\right) \end{aligned}$ |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (1) |  |  |  |  |
|  |  | (2) |  |  |  |  |
|  |  | (1) |  |  |  |  |
| STATE MODELCRDS AGGR. | 2.011.99 | (1) | 2.432.46 | (2) | 2.482.47 | $\left(\begin{array}{l}(3) \\ (3)\end{array}\right.$ |
|  |  | (1) |  |  |  |  |
| REGION MODELI | 1.96 |  |  | (2) |  |  |
| CROS AGGR. |  | (1) | 2.29 | (?) | 2.33 | (3) |
| STATES AGGR.I | 2.00 | (1) | 2.28 | (2) | 2.44 | (3) |



Figure 3. Number indicates the soybean model(s) with smallest percent of test years (1970-1979) having absolute value of the relative difference greater than ten percent. Darker shades indicate CRDs with higher production.



Figure 4. Number indicates the soybean model with smallest value of the largest absolute relative difference during the test years 1970-1979. Darker shades indicate CRDs with higher production.


TABLE
MOUEL COMPARISON GASED OV THE NEXT LAQGEST IRELATIVE DIFFEREVCEI

STRAW MAN MODELS - SOYBEANS
IOWA. ILLINOIS. INOIANA

| STATE CRD | $\text { PD I.INE }{ }_{\text {RAVK }}$ |  | $\begin{aligned} & \text { MODEL } \\ & \text { RDLINES } \\ & \text { RANK } \end{aligned}$ |  | $R 0^{3}$ | $\begin{aligned} & \text { INEC } \\ & \text { RANAK } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| InWA 10 |  |  |  |  |  |  |
| INWA 20 | 77.4 | (1) | -11:0 | (3) | -10.2 | (2) |
| 30 40 | -14.4 | (1) | - 20.0 | (2) | -20.0 | (2) |
| 50 | 16.2 | (2) | -13.6 | (1) | -16.4 | (3) |
| 50 | 11.9 | (1) | -12.7 | (2) | -13.0 | (3) |
| 70 80 | -13.1 | (3) | 10.6 | (1) | -10.8 | $(2)$ |
| 80 90 | -12.7 | (1) | 17.5 19.1 | $\left(\begin{array}{l}\text { (2) } \\ \\ \\ \end{array}\right.$ | 19.0 18.6 | $(3$ |
| STATE MODEL |  | (2) | -9.7 |  | -13.9 |  |
| CODS AGGR. | 12.5 | (3) | -10.8 | (1) | -11.6 | $(3)$ |
|  |  |  |  |  |  |  |
| ILLINOIS10 <br> 20 <br>  <br>  | -12.8 -16.6 | (1) | -17.8 | (3) | -16.1 | (2) |
| 30 | -10.6 | (1) | -14.5 | (2) | -16.0 | (1) |
| 40 | - -9.6 | (1) | -14.4 | (2) | 17.2 | (3) |
| 50 | -12.5 | (1) | 15.4 | (2) | -15.5 | (3) |
| 60 | -10.3 | (1) | -12.0 | (2) | -15.8 | (3) |
| 70 80 | -12.4 -13.9 | (1) | -1790 | (2) | -18.8 | $(2)$ |
| 90 | -19.9 -19.9 | (12) | -17.9 | (1) | -16.0 | (3) |
| STATE MODF CRDS AGGR. | -10.9 -11.7 | (1) | -12.3 | (2) | -15.6 -14.5 | (3) |
|  |  |  |  |  |  |  |
| INDIANA 10 | -13.4 | (1) | 17.3 | (2) | 20.8 | (3) |
| Indina 20 | -15.5 | (1) | -18.9 | $(3)$ | -15.9 | (2) |
| 30 | -18.9 | (2) | -17.3 | (1) | -19.7 | (2) |
| 50 | -12.0 | (1) | 20.3 | (3) | 21.3 | (3) |
| 60 | -10.8 | (1) | 22.7 | (3) | 22.2 | (2) |
| 70 80 | 16.7 | (3) | $-13.5$ | (1) | 14.7 | (2) |
| 80 90 | 27.6 30.4 | (3) | 20.9 22.3 | (1) | 20.2 | (2) |
|  |  |  |  |  |  |  |
| STATEM MONE | -12.0 -11.6 | (1) | 16.7 | (3) | 19.7 | (2) |
|  |  |  |  |  |  |  |
| REGION MODEL | -8.1 | (1) | -13.3 -11.7 | (2) | -13.3 | (2) |
| STATES AGGR. | -8.1 | (1) | -10.9 | (2) | -12.1 | $(3)$ |

Figure 5. Number indicates the soybean model with smallest value of the next largest absolute relative difference during the test years 1970-1979. Darker shades indicate CRDs with higher production.


In 16 of 27 CRDs ( $59 \%$ ), the one line model has the smallest value of the largest absolute relative difference (Figure 4). It also has the smallest value at the state and region levels. The three line model performs worst.

In 19 of 27 CRDs ( $70 \%$ ), the one line model has the smallest value of the next largest absolute relative difference (Figure 5). Except for Iowa, the one line model also ranks first at the state and region levels. The three line model performs worst at the CRD level and is somewhat worse than the two line model at the state and region levels.

## Indicators of Yield Reliability Based on $Y$ and $\hat{Y}$ Show the One Line Model Performing Somewhat Better

Plots of the actual and predicted yields over the ten-year test period for each state model are displayed in Figures 6-8. The model values and the comparative ranks for the indicators of yield reliability based on $Y$ and $\hat{Y}$ are given in Tables 8, 9 and 10. These indicators demonstrate the correspondence between actual and predicted yields. The best performing model will have the largest value for the percent of years in which the direction of change from the previous year and from the average of the previous three years in the $\hat{Y}$ 's agrees with the $Y$ 's and for the correlation coefficient between the actual and predicted yields.

In terms of correctness in direction of change from the previous year, there is little difference in model performance at any level. Figure 9 shows the highest ranking model(s) for each CRD.

When considering correctness of change from the average of the three previous years, the three line model does worst at the CRD level with little difference between the one and two line models (Figure 10). However, the one line model ranks highest at the state and region levels.

In 21 of 27 CRDs (78\%), the Pearson correlation coefficient is closest to +1 for the one line model (Figure 11). The one line model also ranks first at the state and region levels. The three line model performs worst at all levels.

Statistical Tests Based on $\mathrm{d}=\hat{\mathrm{Y}}-\mathrm{Y}$
Show Some Preference for the One Line Model
The results of the parametric and nonparametric paired-sample statistical test are given in Tables 11, 12 and 13. The results for the comparison of the two multi-line models are given in Table ll. There is little evidence of statistically significant differences between the models.

In only one CRD do the parametric test results show a significant difference between the 2 and 3 line models. The 3 line model performs better in that CRD. However, the 2 line model performs significantly better in the Iowa and Illinois state models and the regional results aggregated from the state models.

The more reliable model in each CRD according to the average value of $|d|$ is presented in Figure 12. The 2 line model performs better in 15 of 27 CRDs ( $56 \%$ ), while the 3 line model performs better in 11 of 27 CRDs ( $41 \%$ ), and the

FIGURE 6
IOWA
State Model
Actual and Predicted Yields for the Test Years 1970-1979
STRAWMAN MODELS
SOYBEAVS
= OREDICTED YITUAL YIELD
$1=$ YIELD FOR I LINE MODEL
$2=$ RREDICTES YIELD FOR 2 INE MODEL
$3=$ OREDTCTED YIELD FOR 3 LINE MODEL


FIGURE 7

ILLINOIS
State Model
Actual and Predicted Yields for the Test Years 1970-1979



FIGURE 8

INDIANA
State Model
Actual and Predicted Yields for the Test Years 1970-1979




Figure 9. Number indicates the soybean model(s) with largest percent of test years (1970-1979) having agreement in direction of change from the previous year between predicted and actual yields. Darker shades indicate CRDs with hiqher production.



Figure 10. Number indicates the soybean model (s) with largest percent of test years (1970-1979) having agreement in direction of change from the previous three year average between predicted and actual yields. Darker shades indicate CRDs with higher production.


TABLE 10
UODEL COMDARISON HASED ON THE CORRELATIOV BETWEFV ACTUAL AND PREDICTEO YIELDS DURING I NDFPENOENT TEST YEARS

STOAW MAN MODELS - SOYBEANS IOWA. ILLINJIS. INDIANA

| STATE CRD | R LINE QANK |  | $\begin{aligned} & \mathrm{R}^{\text {MONFL }} \mathrm{C} \text { RINFS } \end{aligned}$ |  | $R^{3 \text { LINEG }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IOWA $\begin{array}{ll}10 \\ & 20 \\ & 30 \\ & 40 \\ & 50 \\ & 60 \\ & 70\end{array}$ |  |  |  |  |  | $\left(\begin{array}{l}2 \\ 3 \\ 3 \\ 2\end{array}\right.$ |
|  | 0.46 | (1) | 0.10 | (3) | 0.25-0.280.21 |  |
|  | $0 \cdot 37$ | (1) | 0.17 | (2) |  |  |
|  | 0.51 | (1) | 0.10 | (3) |  |  |
|  | $0 \cdot 39$ | (1) | $0.0 ?$ | (3) | 0.23-0.24 | (3) |
|  | $-1.12$ | (2) | -0.12 | (1) |  |  |
|  | 0.25 | (1) | -0.32 | (3) | -0.08 | (3) |
|  | -0.43 | (3) | -0.15 | (1) | -0.24 (2) |  |
|  | -0.49 | (3) | - 11.36 | (1) | -1) +3 ( 2 ) |  |
|  | 0.13 | (1) | -0.21 | (2) | -0.34 | (3) |
| $\begin{aligned} & \text { STATE NODEL } \\ & \text { CRDS } \triangle G G R . \end{aligned}$ | 0.34 | (1) | 0.94 | ( 3 ) | -0.06 | $\begin{aligned} & (3) \\ & (3) \end{aligned}$ |
|  | 0.33 | (1) | -1).05 | (7) | -0.06 |  |
|  |  |  |  |  |  |  |
| ILLINOIS $\begin{gathered}10 \\ \\ \\ 20 \\ 30 \\ \\ 40 \\ \\ \\ \\ \\ \\ 60 \\ \\ 70 \\ \\ \\ \\ \\ \\ \\ \\ \end{gathered}$ | 0.14 | (1) | -0.27 | (2) | -0.44-0.03 | (3) |
|  | 0.28 | (1) | -0.06 | (?) |  |  |
|  | 0.04 | (1) | - $0 \cdot 37$ | (3) | -0. 32 | (2) |
|  | 0.05 | (1) | -0. 27 | (2) | -0.54 (3) |  |
|  | 0.08 | (1) | -0.23 | (2) | -0.41 (3) |  |
|  | 0.23 | (1) | -i).12 | (3) | 0.01 (2) |  |
|  | 1).48 | (1) | 0.30 | ( 2 ) | 0.12 | (3) |
|  | 0.30 | (1) | 0.29 | (?) |  | $\begin{array}{rr}0.21 & (3) \\ -0.05 & (3)\end{array}$ |  |
|  | 0.07 | (2) | 0.21 | (1) |  |  |  |
| STATE MODEL.CPDS AGGR. |  |  |  |  | $\begin{aligned} & -0.42 \\ & -0.24 \end{aligned}$ |  |
|  | $\begin{aligned} & 0.20 \\ & 0.19 \end{aligned}$ | (1) | 0.02 -0.10 | (2) |  | $\left(\begin{array}{l} 3 \\ (3) \\ (3) \end{array}\right.$ |
|  |  | (1) | -0.10 | (2) |  |  |
| INDIANA $\begin{array}{ll}10 \\ & 20 \\ & 30 \\ & 40 \\ & 50 \\ & 60 \\ & 70 \\ & 80 \\ & 90\end{array}$ | 0.36 |  | -0.00 |  |  |  |
|  | 0.30 | (1) | -0.08 | ( 2 ) | -0.15 |  |
|  | 9.48 | (1) | 1). 27 | (2) |  |  |  |
|  | 0.18 | (1) | -0. 0.33 | (3) | $\begin{array}{r} 0.16 \\ -0.25 \end{array}$ | (3) (2) |
|  | 0.47 | (1) | 0.1 ? | (3) | 0.16-0.20 | $\left(\begin{array}{l}2 \\ 3\end{array}\right.$ |
|  | $0 \cdot 44$ | (1) | -0. 0 ? | (2) |  |  |
|  | 0.53 | (1) | 0.34 | (3) | -0.45 | (2) |
|  | -0.74 | (3) | -0.11 | (1) | -0.27 | (2) |
|  | -0.77 | (3) | -0.04 | (7) | -0.03 |  |
| STATE MODEL CRDS AGGR. | $\begin{aligned} & 0.44 \\ & 0.45 \end{aligned}$ | (1) | 0.15 | $(2)$ | $\begin{aligned} & 0.04 \\ & 0.07 \end{aligned}$ | $(3)$$(3)$ |
|  |  | (1) | 0.79 | ( 2 ) |  |  |
|  |  |  |  |  |  |  |
| $\begin{array}{cc} \text { REGION MODEL } \\ \text { CROS AGGR. } \\ \text { STATES AGGR. } \end{array}$ | $\begin{aligned} & 0.38 \\ & 0.35 \\ & 0.33 \end{aligned}$ |  |  |  | $\begin{aligned} & -0.21 \\ & -0.05 \\ & -0.15 \end{aligned}$ | $\left(\begin{array}{l} 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{array}\right)$ |
|  |  | (1) | -11. 14 | ( 2 ) |  |  |
|  |  | (1) | 0.01 | (?) |  |  |
|  |  | (1) | 0.05 | ( 2 ) |  |  |

Figure 11. Number indicates the soybean model with the largest correlation coefficient between actual and predicted yields over the test years (1970-1979). Darker shades indicate CRDs with higher production.

models are tied in 1 CRD. The 2 line model performs better in more of the high producing CRDs.

The nonparametric test results show somewhat more statistical difference between the two models. Significant differences between the models are observed in four CRDs. Three of these favor the 2 line model. The 2 line model performs significantly better in the Iowa and Illinois state models and in the regional results aggregated from the state models. The 2 line model also performs significantly better in the Illinois results aggregated from the CRDs and in the region model.

The better model in each CRD according to the percent of years with smaller $|\mathrm{d}|$ is presented in Figure 13. The 2 line model performs better in 12 of 27 CRDs ( $44 \%$ ), the 3 line model performs better in 10 of 27 CRDs ( $37 \%$ ) and the models are tied in 5 CRDs. The 2 line model performs better in more of the high producing CRDs.

In summary, the results of the statistical tests for yield reliability indicate that the 2 line state model performs better than the 3 line model in Iowa and Illinois. No conclusion can be drawn for Indiana.

The results for the comparison of the 1 line model with the 2 line model are given in Table 12. Again, there is little evidence of statistically significant differences between the models.

The parametric test results show a significant difference in three CRDs, favoring the 1 line model over the 2 line model in each case. There are no significant differences between the models at the state or region level.

The better model in each CRD according to the average value of $|d|$ is presented in Figure 14. The 1 line model performs better than the 2 line model in 24 of 27 CRDs ( $89 \%$ ), including the high producing CRDs.

The nonparametric test results again show somewhat more statistical difference between the two models. Seven CRDs show significant differences between the models, all favoring the 1 line model. The 1 line model also performs significantly better in the Iowa results aggregated from the CRDs.

The better model in each CRD according to the percent of years with smaller $|\mathrm{d}|$ is presented in Figure 15. The 1 line model performs better in 19 of 27 CRDs ( $70 \%$ ), the 21 ine model performs better in 3 of 27 CRDs ( $11 \%$ ) and the models are tied in 5 CRDs. The 1 line model performs better in more of the high producing CRDs.

In summary, the 1 line model consistently performs slightly better than the 2 line model at the CRD level. Consequently, the state and region results aggregated from the CRDs favor the 1 line model, although usually not by enough to be declared statistically significant. The Iowa and Indiana state models and the region model favor the 1 line model, but again, the difference is not statistically significant.

The results for the comparison of the 1 line model with the 3 line model are given in Table 13. Again, there is little evidence of statistically significant differences between the models.


Figure 12. Comparison of 2 line and 3 line models to predict soybean yields based on the average of $|d|=|\hat{Y}-Y|$ for 1970-1979. Number indicates model with smaller average $|\mathrm{d}|$. Blank denotes tie. Stars indicate the level of significance, none ( $\mathrm{P}>0.10$ ) . * $(0.05<\mathrm{P}<0.10), * *(0.01<\mathrm{P}<0.05), * * *(\mathrm{P}<0.01)$. Darker shades indicate CRDs with higher production.


Figure 13. Comparison of 2 line and 3 line models to predict soybean yields based on the percent of test years (1970-1979) with smaller $|\mathrm{d}|=|\hat{\mathrm{Y}}-\mathrm{Y}|$. Number indicates model with larger percent. Blank denotes tie. Stars indicate the level of significance, none ( $\mathrm{P}>0.10$ ), * $0.05<\mathrm{P}<0.10$ ), ** ( $0.01<\mathrm{P}<0.05$ ), *** $(\mathrm{P}<0.01)$. Darker shades indicate CRDs with higher production.


TARLE I?
WONEL COMOANISON BASEO OV
PAIRED=SAMPLE STATISTICAL TESTS


STUAN MAN MODELS - SOYGEAVS
IONA ILLINIS INOIANA


Figure 14. Comparison of 1 line and 2 line models to predict soybean yields based on the average of $|\mathrm{d}|=|\hat{\mathrm{Y}}-\mathrm{Y}|$ for 1970-1979. Number indicates model with smaller average $|a|$. Blank denotes tie. Stars indicate the level of significance, none $(P>0.10), *(0.05<P<0.10), * *(0.01<P<0.05)$, $* * *(P<0.01)$. Darker shades indicate CRDs with higher production.


Figure 15. Comparison of 1 line and 2 line models to predict soybean yields based on the percent of test years (1970-1979) with smaller $|\mathrm{d}|=|\hat{Y}-Y|$. Number indicates model with larger percent. Blank denotes tie. Stars indicate the level of significance, none $(\mathrm{P}>0.10), *(0.05<\mathrm{P}<0.10), * *(0.01<\mathrm{P}<0.05)$, *** $(\mathrm{P}<0.01)$. Darker shades indicate CRDs with higher production.


TARLF 13
MODEL COMPARISON BASED OV ( $\%$ LINE MOOEL WITH 3 LINE YOSEL

STRAW MAN MODELS - SOYBEAVS IOWA, ILLINOIS. INOTAṼA


Figure 16. Comparison of 1 line and 3 line models to predict soybean yields based on the average of $|\mathrm{d}|=|\hat{\mathrm{Y}}-\mathrm{Y}|$ for 1970-1979. Number indicates model with smaller average $|d|$. Blank denotes tie. Stars indicate the level of significance, none ( $\mathrm{P}>0.10$ ) , * ( $0.05<\mathrm{P}<0.10$ ), ** ( $0.01<\mathrm{P}<0.05$ ), *** $(\mathrm{P}<0.01)$. Darker shades indicate CRDs with higher production.


Figure 17: Comparison of 1 line and 3 line models to predict soybean yields based on the percent of test years (1970-1979) with smaller $|d|=|Y-Y|$. Number indicates model with larger percent. Blank denotes tie. Stars indicate level of significance, none ( $\mathrm{P}>0.10$ ), * ( $0.05<\mathrm{P}<0.10$ ), ** ( $0.01<\mathrm{P}<0.05$ ), *** $(\mathrm{P}<0.01)$. Darker shades indicate CRDs with higher production.


The parametric test results show a significant difference in only one CRD. It favors the 1 line model over the 3 line model. The 1 line model performs significantly better in the Iowa state model and the Iowa results aggregated from the CRDs.

The better model in each CRD according to the average value of $|d|$ is presented in Figure 16. The 1 line model performs better than the 3 line model in 24 of 27 CRDs ( $89 \%$ ), including the high producing CRDs.

The nonparametric test results again show somewhat more statistical difference between the models. Four CRDs show significant differences between the two models, all favoring the 1 line model. The 1 line model also performs significantly better in the Iowa and Illinois state models, the lowa results aggregated from the CRDs, and the region model.

The better model in each CRD according to the percent of years with smaller $|d|$ is presented in Figure 17. The 1 line model performs better in 16 of 27 CRDs (59\%), the 3 line model performs better in 7 of 27 CRDs ( $26 \%$ ) and the models are tied in 4 CRDs. The 1 line model performs better in more of the high producing CRDs.

In summary, the 1 line model of ten performs slightly better than the 3 line model at the CRD level. Consequently, the state and region results aggregated from the CRDs favor the 1 line model, although usually not by enough to be declared statistically significant. The Iowa and Illinois state models and the region model favor the 1 line model, significantly so for both the parametric and nonparametric tests in lowa. The nonparametric results were significant for the Illinois state model and the region model. The Indiana parametric results favor the 1 line model, but the nonparametric results show no difference between the models.

## A11 Models Are Objective

No subjective inputs are required to run any of these models. In all three cases, the single independent variable is objectively defined as year minus 1950. The join point (s) for the multi-line models as well as the parameter estimates for all models are objectively determined using least squares algorithms.

## Al1 Models Omit Consideration <br> of Known Scientific Relationships

The straw man models do not consider factors which have a recognized causal relationship with crop yields. For example, it is well known that year-to-year variations in weather have an important effect on yield. Therefore, if weather data were available, it would be consistent with scientific knowledge to include weather variables in a model predicting crop yields. Weather variables are excluded from the straw man models yet nothing is done to account for the fact that the yields have been influenced by weather. The yields may also have been influenced by other non-technology factors. However, since no adjustment is made to the yields for these non-technology factors and since these factors are not included as independent variables in the model, the straw man model results will be affected by non-technology influences.

The assumption of straw man model 1 , the simple linear model, is that the rate of change in yields has stayed constant over the model development base period. An attempt was made to investigate the validity of that assumption by fitting multi-line models which allow more than one time trend. However, if the multi-line models do not perform as well as the single line model, one should not necessarily conclude that the assumption of the single line model has been confirmed. Rather the method of accounting for variation in time trends by the multi-line straw man models may not be adequate.

## All Models Are Adequate

The three straw man models require the same input data, year and yield. Therefore, they are equally adequate. They can be developed for any crop growing region or subdivisions thereof and for any special application, such as irrigated yield, as long as the basic inputs of year and yield are available. Likewise they can be developed for areas for which acreage estimates are available in order that production estimates may be obtained.

## All Models Are Timely

As soon as reliable figures are available for the current year's yield, each of the models can be developed and used to produce an estimate of the following year's yield. In this respect, they are all, equally, very timely.

## The One Line Model is Least Expensive <br> but None of the Models Are Costly

The only data required by any of the straw man models are the year and actual yield. These data are readily available at no additional cost.

The one line model can be fit using any standard statistical packaged program or statistical calculator. The multi-line models require the use of a special FORTRAN program which contains the least squares algorithm for objectively estimating the join point (s). Objectively determining two join points takes more computer time than determining a single join point. In summary, the cost of fitting the model parameters is least expensive for the one line model and most expensive for the three line model. However, the cost of fitting any of these models is very low.

## Al1 Models Are Simple

The philosophy behind the straw man models, describing technological trend, is simple. Users can clearly understand the basis for predicted yields. They can also understand the limitations of the models. What a user is to do in interpreting these limitations may not be so simple. The calculation of predicted yields is easy. The $X$ values in the model are simply the year minus 1950. Thus to estimate the yield for 1980 , multiply the slope by 30 and add the intercept. In the case of the multi-line models, the slope and intercept corresponding to the most recent line segment is used.

One cautionary note needs to be made, however. Although the philosophy behind the model is simple, the implementation of the idea may not have been successful. The data has not been adjusted for the effects of weather. An unusually low or high yield related to weather conditions will have an impact, particularly on the multi-line models. This is because their line segment parameters are estimated from shorter time periods and will be influenced more by variations from an overall trend due to an individual year's weather or other non-persistent factors.

All Models Have a Poor Current
Measure of Modeled Yield Reliability
The Spearman correlation coefficient between the estimate of the standard error of a predicted yield from the base period model, $S \hat{Y}$, and the absolute value of the difference between the predicted and actual yield, $|d|$, indicates whether the model provides a useful current measure of modeled yield reliability. An $r$ value close to +1 is desirable since it indicates that a smaller standard error of prediction (and therefore a narrower confidence interval about the predicted value) is associated with smaller discrepancies between predicted and actual yields. If this were the case, one would have confidence in sy as (at least) a relative indicator of the accuracy of $\hat{Y}$. Any model which is primarily a function of trend, such as the straw man models, is not expected to perform well on this criteria using the test described.

From examining Table 14, one can see that most of the correlations are negative for all three models. The results for all three models are so poor that there is no value in making detailed model comparison for this characteristic.

## CONCLUSIONS

All of the straw man models attempt to explain differences in soybean yields over time by fitting trend lines to the yield data. Straw man model 1 , simple linear regression, describes a uniform increase in yield over time. Straw man models 2 and 3 allow the rate of change in yields to vary over the time period. There is little difference between the three models in relation to seven of the eight criteria for model comparison: objectivity, consistency with scientific knowledge, adequacy, timeliness, minimum costs, simplicity, and accurate current measures of modeled yield reliability. Straw man model 1 is somewhat less consistent with scientific knowledge, less costly, and simpler. It performs best in terms of the remaining criteria, yield indication reliability. However, the superior performance is of ten by a small margin and is often not statistically significant.

## RECOMMENDATIONS

Since the largest difference between models is in relation to yield indication reliability and the one line model performs best in regard to that criteria, straw man model 1 is recommended for predicting soybean yields over straw man models 2 and 3. However, this does not necessarily substantiate that the rate of change in yields is uniform over time. More sophisticated trend models still might be able to demonstrate the validity of different rates of change over different portions of the time period considered.

TABLE 14

STRAW MAN MODEIS - SOYBEANS
IOWA, ILLINOIS. INOIANA

| STATE CRJ | $P^{1} \text { LINE } D A N K$ |  | $R^{2} \mathrm{MODEL} \mathrm{NES}$ |  | $p^{3}$ | $\begin{aligned} & \text { NE G } \\ & \text { RANK } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| IOWA | -0.15 | (2) | 0.08 | (1) | -0.31 | (3) |
|  | 0.16 | (1) | 0.08 | (2) | -0.21 | (3) |
|  | -0.05 | (1) | -0.33 | (2) | -0.60 | (3) |
|  | -0.32 | (2) | -0.23 | (1) | -0.51 | (3) |
|  | 0.31 | (1) | -0.34 | (2) | -0.72 | (3) |
|  | -0.05 | (2) | -0.30 | (3) | -0.02 | (1) |
|  | -0. 0.7 | (2) | -0.19 | (1) | -0.62 | $(3)$ |
|  | -0.70 | (3) | -0.5 .3 -0.20 | (1) | -0.66 -0.16 | (2) |
|  | -0.62 | (3) | -0.20 |  | -0.16 | (1) |
| $\begin{aligned} & \text { STATE } A \cap D E L \\ & \text { CRDS } \triangle G G R . \end{aligned}$ | $-0.28$ | (3) | -0.25 | (2) | 0.09 -0.36 | $\left(\frac{1}{3}\right)$ |
|  | $-1) .21$ | (2) | $-0.16$ | (1) | -0.56 | (3) |
|  |  |  |  |  |  |  |
| ILLIVOIS $\begin{array}{r}10 \\ 20 \\ 30 \\ 40 \\ \\ \\ \\ \\ \\ \\ 70 \\ \\ \\ 40 \\ \\ \\ \\ \\ \\ \end{array}$ | -0.0.04 |  | 0.19 | (1) | -0.32 |  |
|  | 0.12 | (1) | 0.02 | (2) | -0.20 | (3) |
|  | -0.48 | (3) | -0.07 | (2) | 0.14 | (1) |
|  | -0.36 | (3) | 0.13 | (1) | -0.09 | (2) |
|  | -0.66 | (3) | 0.01 | (1) | -0.34 | $(2)$ |
|  | -0.62 | (2) | -0.27 | (1) | -0.75 | (3) |
|  | -0.11 | (2) | 0.07 | (1) | -0.28 | (3) |
|  | -0.02 | (3) | -0.43 | (1) | -0.47 | (2) |
|  | -0.71 | (3) | -0.46 | (2) | -0.02 | (1) |
| $\begin{aligned} & \text { STATE MOOEL I } \\ & \text { CRDS AGGR. } \end{aligned}$ |  |  |  | (2) | 0.07 | (1) |
|  | $\begin{array}{r} -1.30 \\ -6.15 \end{array}$ | (2) | $0: 28$ | (1) | -0.18 | (3) |
|  |  |  |  |  |  |  |
| INOI ANA $\begin{array}{ll}10 \\ & 20 \\ & 30 \\ & 40 \\ & 50 \\ & 60 \\ & 70 \\ & 80 \\ & 90\end{array}$ |  |  |  |  |  |  |
|  | -0.33 0.09 | (2) | 0.27 0.10 | (1) | -0.33 -0.09 | (3) |
|  | -0.16 | (?) | -0.41 | (3) | 0.10 | (1) |
|  | -0.10 | (2) | -0.29 | (3) | 0.01 | (1) |
|  | -0.06 | (1) | -0.30 | (3) | -0.18 | (2) |
|  | -0.78 | (3) | -0. 55 | (2) | -0.41 | (1) |
|  | -0.11 | (1) | -0.25 | (3) | -0.11 | (2) |
|  | -0.30 | (1) | -0.4.3 | (3) | -0.35 | (2) |
|  | 0.05 | (1) | -0.60 | (3) | -0.14 | (2) |
| STATE MNOEL CRDS AGGR. |  |  |  |  |  |  |
|  | $\begin{array}{r} -0.21 \\ 0.02 \end{array}$ | $\left(\begin{array}{l} 1 \\ 1 \\ 1 \end{array}\right)$ | -0.48 -0.43 | $\left(\begin{array}{l}3 \\ (3)\end{array}\right.$ | $\begin{aligned} & -0.27 \\ & -0.30 \end{aligned}$ | (2) |
|  |  |  |  |  |  |  |
| RFGION MODEL |  |  |  |  | -0.15 |  |
| RF. GROS AGGR. |  | (2) | 0.53 | (1) | 0.07 | (3) |
| STATES AGGR.I | 0.02 | (3) | 0.35 | (1) | 0.18 | (2) |

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## Measures of Model Performance

## Definition of Terms:

$Y_{i}=$ Yield as reported by U.S.D.A. for year $i$ ("true" or "actual" yield).
$\hat{Y}_{i}=$ Yield as predicted by a model for year $i$.
$d_{i}=\hat{Y}_{i}-Y_{i}=$ difference between predicted and actual yield for year $i$.
$r d_{i}=100 d_{i} / Y_{i}=$ relative difference for year $i$.
$s_{Y_{i}}=$ Standard error of regression $=$ (Residual or Error Mean Square from Model Development Base Period) ${ }^{\frac{1}{2}}$ for year i.
$s_{\hat{Y}_{i}}=$ Standard error of a predicted value for year $i=s_{Y_{i}}\left(1+\underline{x}_{0}{ }^{\prime}\left(\underline{X}^{\prime} \underline{X}\right)^{-1} \underline{x}_{0}\right)^{\frac{1}{2}}$, where $X$ is the regression design matrix of independent variable values and $\underline{x}_{0}$ is the vector of independent variable values for the year the prediction is being made.
$i=1, \ldots, n=$ number of test years and $\Sigma=\sum_{i=1}^{n}=$ summation over the test years.
$\bar{Y}=1 / n \sum_{i=1}^{n} Y_{i}=$ average actual yield.

## Measures:

Bias $=B=1 / n \sum d_{i}=\bar{d}$.

Relative Bias $=R B=100 \mathrm{~B} / \overline{\mathrm{Y}}$.

Mean Square Error $=\operatorname{MSE}=1 / n \Sigma \mathrm{~d}_{\mathrm{i}}{ }^{2}$.
Root Mean Square Error $=$ RMSE $=(\text { MSE })^{\frac{1}{2}}$.

Relative Root Mean Square Error $=$ RRMSE $=100 \mathrm{RMSE} / \overline{\mathrm{Y}}$.
Variance $=\operatorname{Var}=1 / n \Sigma\left(d_{i}-\bar{d}\right)^{2}$.
Standard Deviation $=S D=(\text { Var })^{\frac{1}{2}}$.

Relative Standard Deviation $=\operatorname{RSD}=100$ SD $/(\bar{Y}+\bar{d})$.
Mean Square Error $=$ Variance $+(\text { Bias })^{2}$,
or
Accuracy $=$ Precision $+(\text { Bias })^{2}$.
Pearson $r$ between $\hat{Y}_{i}$ and $Y_{i}$ :

$$
r=\left[\Sigma \hat{Y}_{i} Y_{i}-\frac{\left(\Sigma \hat{Y}_{i}\right)\left(\Sigma Y_{i}\right)}{n}\right]\left[\left(\Sigma \hat{Y}_{i}{ }^{2}-\frac{\left(\Sigma \hat{Y}_{i}\right)^{2}}{n}\right)\left(\Sigma Y_{i}{ }^{2}-\frac{\left(\Sigma Y_{i}\right)^{2}}{n}\right)\right]^{-\frac{1}{2}}
$$

Spearman $r$ between $\left|d_{i}\right|$ and $s_{\hat{\mathbf{Y}}_{i}}$ :
Let $R\left(\left|d_{i}\right|\right)=$ the rank of $\left|d_{i}\right|, R\left(s_{\mathcal{Q}_{i}}\right)=$ the rank of $s_{\hat{Y}_{i}}$, and
$f_{i}=R\left(\left|d_{i}\right|\right)-R\left(s_{\hat{Y}_{i}}\right), i=1, \ldots, n$. Then,
$r=1-\frac{6 \Sigma f_{i}{ }^{2}}{n^{3}-n}$.

## Paired-Sample Statistical Tests Comparing

the Performance of Two Crop Yield Models.

## Definition of Terms:

$\hat{\mathrm{Y}}_{1_{1}}=$ Yield as predicted by model 1 for year 1.
$\hat{\mathrm{Y}}_{\mathbf{2}_{i}}=$ Yield is predicted by model 2 for year 1.
$\left|d_{1_{i}}\right|=\left|\hat{Y}_{1_{i}}-Y_{i}\right|=$ Absolute value of the difference between model 1 predicted and actual yield for year 1.
$\left|d_{2_{i}}\right|=\left|\hat{Y}_{2_{i}}-Y_{i}\right|=$ Absolute value of the difference between model 2 predicted and actual yield for year 1.
$D_{i}=\left|d_{1_{i}}\right|-\left|d_{2_{i}}\right|$.
$\operatorname{Rank}\left(\left|D_{i}\right|\right)=$ Ranks of the absolute values of $D_{i}$ assigned in ascending order (smallest value of $\left|D_{i}\right|=\operatorname{rank} 1, \ldots$, largest value of $\left|D_{i}\right|=$ rank $n$ ). If two or more years have the same value for $\left|D_{i}\right|$, assign each year the average of the ranks.

## Parametric Test - Student t:

$H_{0}: \quad \mu_{D}=0$
$H_{a}: \quad \mu_{D} \neq 0$
Test Statistic $=t=\frac{\overline{\mathrm{D}}}{\mathrm{s}_{\overline{\mathrm{D}}}}$, where

$$
\begin{aligned}
& \bar{D}=1 / n \Sigma D_{i}, \\
& s_{\bar{D}}=\left(s_{D}^{2} / n\right)^{\frac{1}{2}}, \text { and } \\
& s_{D}^{2}=\left[\Sigma D_{i}^{2}-1 / n\left(\Sigma D_{i}\right)^{2}\right] /(n-1) .
\end{aligned}
$$

Reject $H_{0}$ if $|t|>t_{\alpha,(n-1)}$.

## Nonparametric Test - Wilcoxon Signed Rank:

$H_{0}$ : There is no difference in the performance of the models.
$H_{a}$ : There is a difference in the performance of the models.
Procedure to compute test statistic, $T$ :

1. Compute the $\mathrm{D}_{\mathrm{i}}$.
2. Assign ranks to $\left|D_{i}\right|$.
3. Assign signs to Rank $\left(\left|D_{i}\right|\right)$ corresponding to the signs of $D_{i}$.
4. Let $T=$ the absolute value of the sum of the ranks with the less frequent sign.

Reject $H_{0}$ if $T \leq T_{\alpha(1 ~ t a i l e d), ~}^{n}$.





| ADPFVOIX <br> BOOTSTRAP TEST RESULTS FOQ SOYBEAN YIELOS IN <br> IOWA. TLLINOIS: AND INDIANA <br> SM l=STRAW COMPARING STRAWMAN MODELS |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |






| SM | $1=S T R$ |  | $\begin{aligned} & \text { BOTSTR } \\ & \text { FOR SOY } \\ & \text { AMP ILLI } \\ & \text { OMAR } \\ & \text { II SM } \end{aligned}$ | APPENA BEAN NOIS: $2=$ STRA | $\begin{aligned} & \text { IX } \\ & T \text { PES } \\ & \text { IELDS } \\ & \text { AND II } \\ & \text { WMAN } \\ & \text { WAAN } \end{aligned}$ | $\begin{aligned} & \text { LTS } \\ & \text { IN } \\ & \text { IDIANA } \\ & 10 D E L S \\ & 2 \text { SM } \end{aligned}$ | 3=STRAW | MAN | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STATE | CRO | YEAR | $\begin{aligned} & \text { ACTUAL } \\ & \text { YIFLD } \\ & (0 / H) \end{aligned}$ | SM | $\begin{aligned} & \text { EDICT } \\ & \text { LO } \\ & \text { SM } \end{aligned}$ | $\text { SM } 3$ | $\underset{\text { OREDICT }}{ }$ |  | SMAL |
| INDIANA | 50 |  |  |  |  |  |  |  |  |
|  |  | 1971 | 24.1 | 21.2 | 23.1 | 21.9 |  | - 0 | 2 |
|  |  | 1972 | 20.7 | 22.1 | 24.9 | 25.1 | -2. | 4.2 | 4. |
|  |  | 1973 | 23.6 | 22.2 | 24.0 | 24.2 | -1.4 | 0.4 | 0.6 |
|  |  | 1974 | 24.4 | 2.2.? | 24.4 | 22.5 | -2.0 | -2.7 | -6.9 |
|  |  | 1976 | 25.0 | 22.9 | 22.4 | 23.4 | -2.1 | -2. 5 | -1.6 |
|  |  | 1977 1978 1 | 25.9 | $23: 4$ 24.3 | 24.1 25.4 | 24:4 | -3.5 | -2.88 | -2.5 |
|  |  | 1978 1979 | 25.5 | 24.3 | 25.4 | 25:5 | -2.0 | 0.9 2.1 | -0.7 2.3 |
|  | 50 | 1970 | 20.1 | 18.5 | 20. | 20.3 | -1.5 | 0.0 |  |
|  |  | 1971 | 21.2 | $18: 9$ | 21.4 | 21.4 | -2.3 | 0.2 | 0.2 |
|  |  | 1972 | 18.5 | 19.7 | 22.7 21.6 | 22.5 | 0.9 -1.6 | 4.2 | $4 \cdot 1$ |
|  |  | 1974 | 17.2 | 20.4 | 22.0 | 22.4 | -1.6 | 4.8 | 5.6 |
|  |  | 1975 | 22:? | 20.1 | 20.6 | 20: 7 | -2.1 | -1.6 | 1.3 |
|  |  | 1976 | 21.0 | 20.8 | 21.5 | 21.7 | -0.2 | -0.5 | 0:7 |
|  |  | 1978 | 23.1 | 21.7 | 22.6 | 23.7 | -2.1 | - 0.4 | 2.7 |
|  |  | 1979 | 27.0 | 22.3 | 23:1 | 23.4 | -1.4 |  |  |
|  | 70 | 1970 |  |  |  |  |  |  |  |
|  |  | 1971 | 20.4 | 20.9 | 21.5 | 20.1 | $0: 5$ | 1.1 | 0.3 |
|  |  | 1972 1973 | 19.8 18 | 21.4 | 21.8 | 20.5 | $\frac{1}{3} \cdot 6$ | $2 \cdot 0$ | 0.7 |
|  |  | 1973 | 17.7 | 21.7 | 20.9 | 20.5 | 3.1 | 2.3 | 1.9 |
|  |  | 1975 | $21: 3$ | $21: 4$ | 20.1 | 19.5 | 0.1 | -1:2 | -1. |
|  |  | 976 | 23.7 | 21.7 | 20.5 | 19.7 | -2.0 | 3.2 | $4 \cdot 0$ |
|  |  | 1978 | 23.8 | 22.1 | 21.3 | 21.5 | -1.7 | -2.5 | - |
|  |  | 1979 | 23.2 | 22.5 | 24.2 | 23.6 | -0.? | -0.2 | . 2 |
|  | 80 |  |  |  |  |  |  |  |  |
|  |  | 1971 | $19: 2$ | 20.3 | 20.9 | 19.7 | 1.1 | 1.4 | 0.7 |
|  |  | 1972 1973 | 17:6 | 20.8 | 21.1 | 20.0 | 3. 2 | 3:5 | 2.4 |
|  |  | 1974 | 16.6 | 20.5 | 18.9 | 19.1 | 4.9 | 2.4 | 2. 5 |
|  |  | 1975 | 15.1 | 20:? | 18.3 | 18.2 | $5: 1$ | 3 3:2 | 3.1 |
|  |  | 1976 | 19.6 | 19.6 | 18.0 | 17.7 | 0.0 | -1.6 | 1.9 |
|  |  | 1977 1978 | 20.8 19.9 | 19.5 | 18.0 | 17.9 | - 1.3 | -2.8 | . 9 |
|  |  | 1979 | 19.0 | 19.5 | 18.3 18.6 | 18.5 | -0.2 | -1.6 | 1.5 |


|  |  | Ncosiorintmmax ómмóominio <br> ＝minnarts． n－Mociñó <br> nognemininx <br>  <br> NNTMni－ston がずがかかのどか <br> かOMnM－o－x <br>  <br>  <br> －ingoinnoninm －incoooco nNTuNNM－NNT <br> ONANJOJNN <br>  <br>  <br> にーNMJUNONの －Nrararrar ののaのaのaかのa <br> o <br> $\sum$ 2 2 2 2 | innomn－asnout oimmininioo <br> rancs monns． ocmmiñーico 11111 <br> －arnorinmoinn <br>  <br> movinintinman －nmñíanim <br>  <br> osicninismatan －••••••••• <br>  <br> －jonincsunc <br>  NNNNNNTNNN <br> minaruminatun onv゙ーがnviさm゙ nnmamninNun <br> －－nNMUNONかの rantrarrar oanoanooan <br>  <br> $\stackrel{\leftrightarrow}{\sim}$ | nonmarinnuat pimーinMoo <br> ssmanamaran iominninioc <br>  <br>  <br> nsennotrnn <br>  <br>  <br>  ominio－nint NNNNNINNVNT <br> maminarexa <br>  n．NNNNNNNNN <br> かnonolnagnn onósinまMs <br>  <br> prinmuincuma anNNNNNNA <br>  <br> $\dot{\alpha}$ $\underset{0}{0}$ 0 <br> $\underset{\sim}{\sim}$ |
| :---: | :---: | :---: | :---: | :---: |


| SMA | I=STRAW MA |  | $\begin{aligned} & \text { BOOTSTR } \\ & \text { FOR SOY } \\ & \text { AA ILL } \\ & \text { OMP ARIN } \\ & \text { I SM } \end{aligned}$ | APPFVI $\triangle P$ TES BEAN NOIS. $\mathrm{Z}=\mathrm{STPA}$ $2=S T R A$ | IELOS ANO NMAN $\checkmark$ MAN | $\begin{aligned} & \text { IN } \\ & \text { OIANA } \\ & \text { ODELS } \\ & \text { C SU } \end{aligned}$ | $3=$ STRAW MAN 3 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CRD | YEAR | $\begin{aligned} & A C T U A L \\ & Y I F L I) \\ & (0 / H) \end{aligned}$ | SM ${ }_{\text {Y/ }}^{\text {PR }}$ |  | C) 5 M 3 | $\underset{S A 1}{2 R E D I C T}$ | $\begin{aligned} & D= \\ & E D-A C \\ & S M 2 \end{aligned}$ | SUAL ${ }_{\text {SM }}$ |
| QEGION | MOOEL | 1970 | 21.2 | 21.0 | 21.9 | 21.8 | -0. 2 | 0.7 | 0.6 |
|  | MOローL | 1971 1972 | 220.1 | $21: 3$ 21.8 | 22.3 23.0 | 22.5 23.3 | -0.8 | $0 \cdot ?$ | 0. 5 |
|  |  | 1973 | 21.8 | 22.3 | 23.5 | 23.4 | - 0.5 | 1.7 | 2.0 |
|  |  | 1974 | 17.4 | 22.6 | 23.3 | 23.5 | 5.2 | 5.9 | 6.1 |
|  |  | 1975 | 23.4 | 22.? | 21.2 | 21.1 | -1.2 | -2.? | -2. ${ }^{-2}$ |
|  |  | 1976 | 21.8 | 22.7 | 22:3 | 21.4 | -0.9 | 0.5 | -0.4 |
|  |  | 1977 | 24.8 | 22.8 | 21.5 | 21.5 | -2.0 | -3.3 | -3.3 |
|  |  | 1978 | 23.7 | 23.4 | 23.2 | 23.4 | -0.3 | -0.5 | -0.3 |
|  |  | 1979 | 25:4 | 23.6 | 23.8 | 23:3 | -1.8 | -1.6 | -2.1 |
| CROS | AGGR. | 1970 |  |  |  |  | -0.2 | 0.1 | 0.5 |
|  | AGGR. | 1971 | 22.1 | $21: ?$ | 22.3 | 22.2 | -0.9 | $0 \cdot 2$ | 0.1 |
|  |  | 1972 | 22:8 | $21: 7$ | 22.8 | 22.8 | -1.1 | $0 \cdot 7$ | $0 \cdot 0$ |
|  |  | 1973 | 21.8 | 2?.3 | 23.6 | 23.6 | 0.5 | 1.8 | 1.8 |
|  |  | 1974 | 17.4 | 22.6 | 23.3 | 23.4 | 5.2 | 5.9 | 6.0 |
|  |  | 1975 | 23.4 | 2?.1 | 21.4 | 21.3 | -1.3 | -2.0 | $2 \cdot 1$ |
|  |  | 1976 | 21.8 | 22:7 | 22.1 | 22.0 | 0.9 | 0.3 | $0 . ?$ |
|  |  | 1977 | 24.8 | 22.7 | 21.9 | 21.7 | -2. 1 | -2.9 | -2.9 |
|  |  | 1978 | 23.7 | 23.4 | 23.3 | 23.3 | -0.3 | -0.4 | -0.4 |
|  |  | 1979 | 25.4 | 23.5 | 24.0 | 24.0 | -1.8 | -1.4 | -1.4 |
| StATES | AGGR. | 1970 | 21.2 | 21.0 | 21.0 | 21.7 | -0.2 | -0.2 | 0.5 |
|  |  | 1971 | 22.1 | 21.2 | 22.4 | 22.0 | -0.9 | 0.3 | 0. |
|  |  | 1972 | 22.8 | 21.7 | 22.7 | 22.8 | -1.1 | -0.1 | 0 - 0 |
|  |  | 1973 | 21.8 | 22.3 | 23.7 | 23.9 | 0.5 | 1.9 | 2.1 |
|  |  | 1974 | 17.4 | 2?.7 | 23.4 | 23.5 | $5 \cdot 3$ | 6.9 | 6.1 |
|  |  | 1975 | 23.4 | $22 \cdot 1$ | 21.5 | 21.1 | -1.3 | -1.9 | -2.3 |
|  |  | 1976 | 21.8 | 22.9 | 22.0 | 21.5 | 1.0 | $0 \cdot 2$ | -0.2 |
|  |  | 1977 | 24.8 | 22.9 | 22.1 | 21.9 | -2.0 | -2.7 | -3.0 |
|  |  | 1978 | 23.7 | 23.4 | 23.4 | 23.2 | -0.3 | -0.3 | -0.5 |
|  |  | 1979 | 25.4 | 23.7 | 24.1 | 23.5 | -1.7 | -1.3 | -1.8 |

